Cycle Related Analytic Mean Cordial Graph

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Abstract – Let G= (V,E) be a graph with p vertices and q edges. A Analytic Mean Cordial Labeling of a Graph G with vertex set V is a bijection from V to {-1,1} such that edge uv is assigned the label $|f(u) - f(v)|/2$ with the condition that the number of vertices labeled with -1 and the number of vertices labeled with 1 differby atmost 1 and the number of edges labeled with 1 anad the number of edges labeled with 0 differ atmost 1.The graph that admits a Analytic Mean Cordial Labelling is called Analytic Mean Cordial Graph. In this paper, we proved that cycle related graphs Globe-Gl $_{(n)}$,Flower-$fl_{(n)}$,$(C_3 \times C_3)_n$, $C_3 \odot K_{1,n}$, Double Triangular Snake - $C_2(P_n)$, Quadrilateral Snake - $Q_n$are Analytic Mean Cordial Graphs.

Index Terms – Analytic Mean Cordial Graph, Analytic Mean Cordial Labeling.

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1. INTRODUCTION

A Graph G is a finite nonempty set of object called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair e = {u,v} of vertices in E is called edges or a line of G.In this paper, we proved that cycle related graphs Globe-Gl $_{(n)}$, Flower-$fl_{(n)}$,$(C_3 \times C_3)_n$, $C_3 \odot K_{1,n}$, Double Triangular Snake - $C_2(P_n)$, Quadrilateral Snake - $Q_n$are Analytic Mean Cordial Graphs. For graph theory terminology, we follow [2].

2. PRELIMINARIES

Let G = (V,G) be a graph with p vertices and q edges. A Analytic Mean Cordial Labeling of a Graph G with vertex set is a bijection from V to {-1,1} such that edge uv is assigned the label $|f(u) - f(v)|/2$ with the condition that the number of vertices labeled with -1 and the number of vertices labeled with 1 differ by atmost 1 and the number of edges labeled with 1 and the number of edges labeled with 0 differ atmost 1.

The graph that admits a Analytic Mean Cordial Labeling is called Analytic Mean Cordial Graph. In this paper, we proved that cycle related graphs are Analytic Mean Cordial Globe-$Gl_{(n)}$ Flower-$fl_{(n)}$, $(C_3 \times C_3)_n$, $C_3 \odot K_{1,n}$, Double Triangular Snake - $C_2(P_n)$, Quadrilateral Snake $- Q_n$ Graphs.

Definition:2.1

$(C_3 \times C_3)_n$ is a graph by joining C_n by an edge. Note that $(C_3 \times C_3)_n$ has mn+m-1 edges and mn vertices.

Definition:2.2

Graph obtained from a path P_n, by joining each end vertices of an edge with two isolated vertex.It is denoted by $C_2(P_n)$.

Definition: 2.3

A quadrilateral snake Q_n is obtained from a path $(u_1u_2 \ldots\ldots\ldots u_n)$ by joining u_i, u_{i+1} to new the vertices u_{i+1}, w_i respectively. (i.e) every edge of the path is replaced by a cycle C_4.

Definition: 2.4

$C_3 \odot K_{1,n}$ is a graph obtained by joining each vertex of a star having n edges, to one of the vertex of a cycle of length 3.

Definition: 2.5

Flower is a graph obtained from Helm by joining the pendant vertex to the centre of the wheel. It is denoted by $fl_{(n)}$.

Definition:2.6

Globe is defined as the two isolated are joined by n path of length 2.It is denoted by $Gl_{(n)}$.

3. MAIN RESULT

THEOREM: 3.1

Globe-Gl $_{(n)}$ is Analytic Mean Cordial Graph.

Proof:

Let G be Gl$_{(n)}$

Let V (G) = { u, v , u_i : 1 ≤ i ≤ n}

Let E (G) = {[(uu_i) U (vu_i) : 1 ≤ i ≤ n]}

Define f : V (G) → {-1,1}

The vertex labeling are ,

f(u) = -1

f(v) = 1

$$f(u_i) = \begin{cases} 1 & i \equiv 1 \bmod 2 \\ -1 & i \equiv 0 \bmod 2 \quad 1 \le i \le n \end{cases}$$

The induced edge labeling are,

$$f^*(uu_i) = \begin{cases} 1 & i \equiv 1 \bmod 2 \\ 0 & i \equiv 0 \bmod 2 \qquad 1 \le i \le n \end{cases}$$

$$f^*(vu_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \quad 1 \le i \le n \end{cases}$$

Here,

When n = 2m, m > 0

$v_f(1) = v_f(-1) = n$ for all n and

$e_f(1) = e_f(0) = n$ for all n

When n = 2m + 1, m > 0

$v_f(1) = m + 2$, $v_f(-1) = m + 1$

$e_f(1) = e_f(0) = n$ for all n

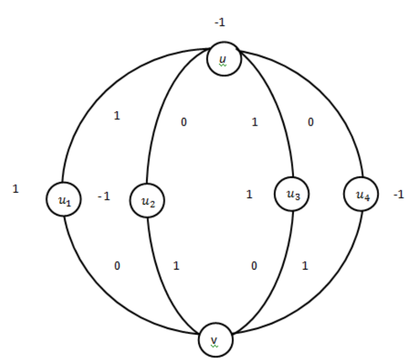
Therefore, Globe- Gl_n satisfies the conditions

$| v_f(1) - v_f(-1) | \le 1$

$| e_f(1) - e_f(0) | \le 1$

Hence, Globe - Gl_n is Analytic Mean Cordial Graph.

For example, The Analytic Mean Cordial Graph Globe - Gl_4 are shown in the figure



THEOREM: 3.3

Flower-$fl_{(n)}$ is Analytic Mean Cordial Graph.

Proof:

Let G be $fl_{(n)}$

Let V (G) = {u, u_i , v_i : $1 \le i \le n$}

Let E (G) = {[(uu_i) U [($u_i v_i$) U (uv_i) U (u_i u_{i+1}) : $1 \le i \le n$]}

Define f : V (G) → {-1, 1}

The vertex labeling are ,

f(u) = -1

$f(u_i)$ = -1 $1 \le i \le n$

$f(v_i)$ = 1 $1 \le i \le n$

The induced edge labeling are,

$f^*(u_i u_{i+1}) = 0$ $1 \le i \le n$

$f^*(uv_i)$ = 1 $1 \le i \le n$

$f^*(v_i u_i)$ = 1 $1 \le i \le n$

$f^*(uu_i)$ = 0 $1 \le i \le n$

Here,

$v_f(1) = n$, $v_f(-1) = n+1$ for all n and

$e_f(0) = e_f(1) = 2n$

Therefore, Flower - $Fl_{(n)}$ satisfies the conditions

$| v_f(1) - v_f(-1) | \le 1$

$| e_f(1) - e_f(0) | \le 1$

Hence, Flower - $Fl_{(n)}$ is Analytic Mean Cordial Graph.

For example, The Analytic Mean Cordial Graph Fl_4 are shown in the figure

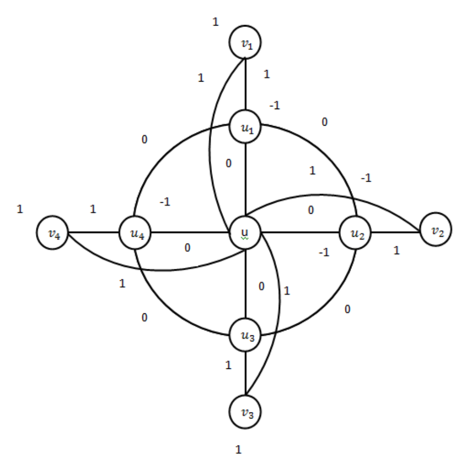


Figure 3.4

THEOREM :3.5

$(C_3 \times C_3)_n$ is Analytic Mean Cordial Graph.

Proof:

Let G be $(C_3 \times C_3)_n$

Let V (G) = { u_i : $1 \le i \le n$, u_{ij} : $1 \le i \le n$ $1 \le j \le 2$ }

Let E (G) = { [$(u_i u_{ij}$):1 ≤ i ≤ n,1 ≤ j ≤ 2] U [$(u_{i1} u_{i2}$):
1 ≤ i ≤ n] U [$(u_{i1} u_{i2}$): 1 ≤ i ≤ n - 1]}

Define f : V (G) → {-1,1}

The vertex labeling are ,

$f(u_i)$ = { -1 i ≡ 0 mod 2 ; 1 i ≡ 1 mod 2 1 ≤ i ≤ n

$f(u_{i1})$ = { -1 i ≡ 0 mod 2 ; 1 i ≡ 1 mod 2 1 ≤ i ≤ n

$f(u_{i2})$ = { -1 i ≡ 1 mod 2 ; 1 i ≡ 0 mod 2 1 ≤ i ≤ n

The induced edge labeling are,

$f^*(u_i u_{i1})$ = 0 1 ≤ i ≤ n

$f^*(u_i u_{i2})$ = 1 1 ≤ i ≤ n

$f^*(u_{i1} u_{i2})$ = 1 1 ≤ i ≤ n

$f^*(u_{i2} u_{(i+1)1})$ = 1 1 ≤ i ≤ n – 1

Here,

When n=4m -2 , m > 0

v_f(1) = v_f (-1) = 6m – 3 and

e_f(0) = 7m-3, e_f(1) = 9m – 6

When n = 4m, m > 0

v_f(1) = v_f (-1) = 6m

e_f(0) = 8m - 1 e_f(1) = 8m

When n = 2m + 1, m > 0

v_f(1) = 3m + 2, v_f (-1) = 3m + 1 and

e_f(0) = 4m + 1 e_f(1) = 4m + 2

Therefore, $(C_3 \times C_3)_n$ satisfies the conditions

| v_f(1) - v_f(-1) | ≤ 1

| e_f(1) - e_f(0) | ≤ 1

Hence $(C_3 \times C_3)_n$ is Analytic Mean Cordial Graph.

For example, The Analytic Mean Cordial Graph $(C_3 \times C_3)_4$ are shown in the figure

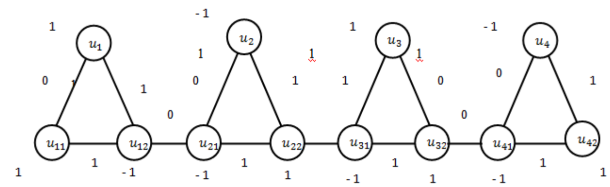


Figure 3.6

THEOREM : 3.7

$C_3 \odot K_{1,n}$ is Analytic Mean Cordial Graph.

Proof :

Let G be $C_3 \odot K_{1,n}$

Let V(G) = { u, v, w,[u_i,: v_i, w_i] : 1 ≤ i ≤ n}

Let E(G) = { (u v) U (v w) U (w u) U [$(u_i u)$ U $(v_i v)$ U (ww):1 ≤ i ≤ n]}

Define f : V(G) → {-1,1}

The vertex labeling are ,

$f(u)$ = 1

$f(v)$ = -1

$f(w)$ = 1

$f(u_i)$ = -1 1 ≤ i ≤ n

$f(w_i)$ = 1 1 ≤ i ≤ n

$f(v_i)$ = { -1 i ≡ 1 mod 2 ; 1 i ≡ 0 mod 2 1 ≤ i ≤ n

The induced edge labeling are,

$f^*(uv)$ = 0

$f^*(uw)$ = 1

f^*(vw) = 1

$f^*(uu_i)$ = 1 1 ≤ i ≤ n

$f^*(ww_i)$ = 1 1 ≤ i ≤ n

$f^*(vv_i)$ = { 0 i ≡1 mod 2 ; 1 i ≡ 0 mod 2 1 ≤ i ≤ n

Here,

When n = 2m , m > 0

v_f(1) = 3m + 2 , v_f (-1) = 3m + 1 and

e_f(0) = 3m + 1, e_f(1) = 3m +2

When n = 2m - 1, m > 0

$v_f(1) = v_f(-1) = 3m$ and

$e_f(0) = e_f(1) = 3m$

Therefore, $C_3 \odot K_{1,n}$ satisfies the conditions

$| v_f(1) - v_f(-1) | \leq 1$

$| e_f(1) - e_f(0) | \leq 1$

Hence $C_3 \odot K_{1,n}$ is Analytic Mean Cordial Graph.

For example, The Analytic Mean Cordial Graph $C_3 \odot K_{1,6}$ are shown in the figure

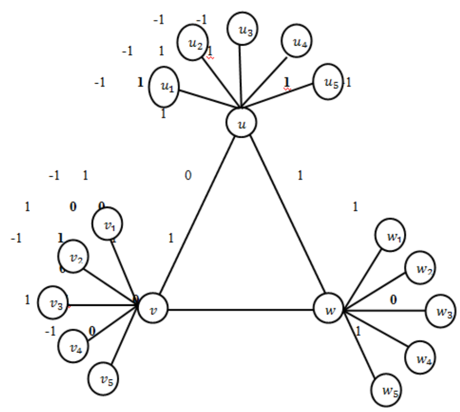


Figure 3.8

THEOREM : 3.9

Double Triangular Snake - $C_2(P_n)$ is Analytic Mean Cordial Graph.

Proof :

Let G be $C_2(P_n)$

Let V(G) = { $u_i : 1 \leq i \leq n$, $v_i, w_i : 1 \leq i \leq n - 1$}

Let E(G) = { [(u_iu_{i+1}) U (u_iv_i) U (u_iw_i) : $1 \leq i \leq n - 1$] U [(u_iv_{i-1}) U (u_iw_{i-1}) : $2 \leq i \leq n$]}

Define f : V(G) → {-1,1}

The vertex labeling are ,

$$f(u_i) = \begin{cases} -1 & i \equiv 2, 3 \bmod 4 \\ 1 & i \equiv 0,1 \bmod 4 \end{cases} \quad 1 \leq i \leq n$$

$$f(w_i) = 1 \quad 1 \leq i \leq n - 1$$

$$f(v_i) = -1 \quad 1 \leq i \leq n - 1$$

The induced edge labeling are,

$$f^*(u_iu_{i+1}) = \begin{cases} 1 & i \equiv 1 \bmod 2 \\ 0 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \leq i \leq n - 1$$

$$f^*(u_iw_i) = \begin{cases} 0 & i \equiv 0,1 \bmod 4 \\ 1 & i \equiv 2,3 \bmod 4 \end{cases} \quad 1 \leq i \leq n - 1$$

$$f^*(u_iv_i) = \begin{cases} 1 & i \equiv 0,1 \bmod 4 \\ 0 & i \equiv 0,1 \bmod 4 \end{cases} \quad 1 \leq i \leq n - 1$$

$$f^*(w_{i-1}u_i) = \begin{cases} 1 & i \equiv 2,3 \bmod 4 \\ 0 & i \equiv 2,3 \bmod 4 \end{cases} \quad 2 \leq i \leq n - 1$$

$$f^*(v_{i-1}u_i) = \begin{cases} 0 & i \equiv 2,3 \bmod 4 \\ 0 & i \equiv 2,3 \bmod 4 \end{cases} \quad 2 \leq i \leq n - 1$$

Here,

When n = 2m , m > 0

$v_f(1) = v_f(-1) = 3m - 1$ and

$e_f(0) = 5m - 3$, $e_f(1) = 5m - 2$

When n = 4m - 1, m > 0

$v_f(1) = 6m - 3$, $v_f(-1) = 6m - 2$ and

$e_f(0) = e_f(1) = 10m - 5$

When n = 4m + 1, m > 0

$v_f(1) = 6m + 1$, $v_f(-1) = 6m$ and

$e_f(0) = e_f(1) = 10m$

Therefore, $C_2(P_n)$ satisfies the conditions

$| v_f(1) - v_f(-1) | \leq 1$

$| e_f(1) - e_f(0) | \leq 1$

Hence, Double triangular Snake - $C_2(P_n)$ is Analytic Mean Cordial Graph.

For example, The Analytic Mean Cordial Graph $C_2(P_4)$ are shown in the figure

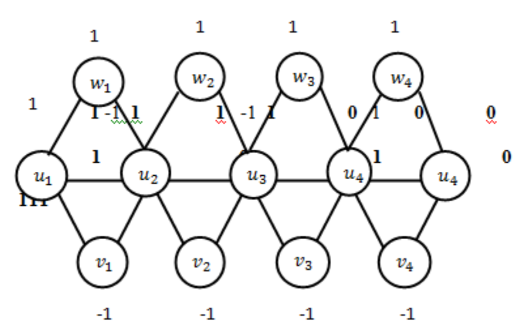


Figure 3.10

THEOREM: 3.11

Quadrilateral Snake - Q_n is Analytic Mean Cordial Graph.

Proof:

Let G be Q_n

Let V(G) = { u_i : 1 ≤ i ≤ n , v_i, w_i : 1 ≤ i ≤ n - 1}

Let E(G) = { [(u_iu_{i+1}) U (u_iv_i) U (u_iw_i) : 1 ≤ i ≤ n -1] U (u_iw_{i-1}) : 2 ≤ i ≤ n]}

Define f : V(G) → {-1,1}

The vertex labeling are ,

$$f(w_i) = \begin{cases} -1 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \le i \le n-1$$

$$f(u_i) = -1 \quad 1 \le i \le n$$

$$f(v_i) = 1 \quad 1 \le i \le n-1$$

The induced edge labeling are,

$$f^*(u_iu_{i+1}) = 0 \quad 1 \le i \le n-1$$

$$f^*(u_iv_i) = 1 \quad 1 \le i \le n-1$$

$$f^*(w_iv_i) = \begin{cases} 1 & i \equiv 0 \bmod 2 \\ 0 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \le i \le n-1$$

$$f^*(w_{i-1}u_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 0 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \le i \le n-1$$

Here,

When n = 2m , m > 0

$v_f(1) = v_f(-1)$ = 3m – 1 and

$e_f(0) = e_f(1)$ = 4m - 2

When n = 2m + 1, m > 0

$v_f(1)$ = 3m , $v_f(-1)$ = 3m + 1 and

$e_f(0) = e_f(1)$ = 4m

Therefore, Q_n satisfies the conditions

| $v_f(1) - v_f(-1)$ | ≤ 1

| $e_f(1) - e_f(0)$ | ≤ 1

Hence, Quadrilateral Snake - Q_n is Analytic Mean Cordial Graph.

For example, The Analytic Mean Cordial Graph Q_4 are shown in the figure.

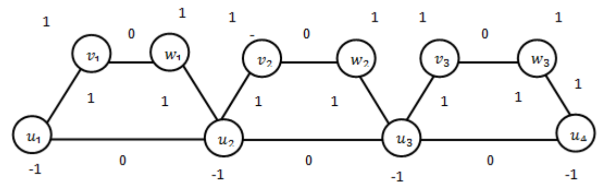


Figure 3.12

REFERENCES

[1] A.Nellai Murugan and V.Baby Suganya, A study on cordial labeling of Splitting Graphs of star Attached C_3 and $(2k+1)C_3$ ISSN 2321 8835, Outreach , A Multi Disciplinary Refreed Journal, Volume . VII, 2014, 142 -147. I.F 6.531

[2] A.Nellai Murugan and V.Brinda Devi , A study on Star Related Divisor cordial Graphs ,ISSN 2321 8835, Outreach , A Multi Disciplinary Refreed Journal, Volume . VII, 2014, 169-172. I.F 6.531

[3] A.Nellai Murugan and M. Taj Nisha, A study on Divisor Cordial Labeling Star Attached Path Related Graphs, ISSN 2321 8835, Outreach , A Multi Disciplinary Refreed Journal, Volume . VII, 2014, 173-178. I.F 6.531

[4] Nellai Murugan and V .Sripratha, Mean Square Cordial Labelling , International Journal of Innovative Research & Studies, ISSN 2319-9725 ,Volume 3, Issue 10Number 2 ,October 2014, PP 262-277.

[5] A.Nellai Murugan and A.L Poornima ,Meanness of Special Class Of Graphs, International Journal of Mathematical Archive. ISSN 2229-5046, Vol 5 , issue 8, 2014, PP 151-158.

[6] A.Nellai Murugan and A.Mathubala, Path Related Homo- cordial graphs, International Journal of Innovative Science, Engineering & Technology , ISSN 2348-7968,Vol.2, Issue 8 ,August. 2015,PP 888-892. IF 1.50,IBI-Factor 2.33

[7] A.Nellai Murugan ,V.Selva Vidhya and M Mariasingam, Results On Hetro- cordial graphs, International Journal of Innovative Science, Engineering & Technology , ISSN 2348-7968,Vol.2, Issue 8 ,August. 2015,PP 954-959. IF 1.50,IBI-Factor 2.33

[8] A.Nellai Murugan , and R.Megala, Path Related Relaxed Cordial Graphs , International Journal of Scientific Engineering and Applied Science (IJSEAS) - ISSN: 2395-3470,Volume-1, Issue-6, September ,2015 ,PP 241-246 IF ISRA 0.217.

[9] A.Nellai Murugan and J.Shiny Priyanka, Tree Related Extended Mean Cordial Graphs, International Journal of Research -Granthaalayah, ISSN 2350-0530,Vol.3, Issue 9 ,September. 2015, PP 143-148. I .F . 2.035(I2OR).

[10] A.Nellai Murugan and S.Heerajohn, Cycle Related Mean Square Cordial Graphs, International Journal Of Research & Development Organization – Journal of Mathematics,Vol.2, Issue 9 , September. 2015, PP 1-11.

[11] A.Nellai Murugan and A.Mathubala, Special Class Of Homo- Cordial Graphs, International Journal Emerging Technologies in Engineering Research, ISSN 2524-6410,Vol.2, Issue 3 ,Oct.2015, PP 1-5.

[12] A.Nellai Murugan , and R.Megala ,Tree Related Relaxed Cordial Graphs, International Journal of Multi disciplinary Research & Development , ISSN: 2349-4182,Volume-2, Issue-10, October ,2015 ,PP 80-84 . IF 5.742.

[13] A.Nellai Murugan , and A.Mathubala ,Cycle Related Homo-Cordial Graphs , International Journal of Multi disciplinary Research & Development, ISSN: 2349-4182,Volume-2, Issue-10, October ,2015 ,PP 84-88 . IF 5.742.

[14] L. Pandiselvi , A. Nellai Murugan , and S. Navaneethakrishnan , Some Results on Near Mean Cordial , Global Journal of Mathematics , ISSN 2395-4760,Volume 4,No : 2,August-2015, PP 420-427.

[15] A.Nellai Murugan and V.Selvavidhya , Path Related Hetro- Cordial Graphs, International Journal Emerging Technologies in Engineering Research, ISSN 2524-6410,Vol.2, Issue 3 ,October 2015, PP 9-14.

[16] A.Nellai Murugan and S.Heerajohn, Special Class of Mean Square Cordial Graphs, International Journal Of Applied Research ,ISSN 2394-7500,Vol.1, Issue 11 ,Part B , October 2015, PP 128-131.I F 5.2
[17] A.Nellai Murugan and J.Shinny Priyanka , Extended Mean Cordial Graphs of Snakes ,International Journal Emerging Technologies in Engineering Research, ISSN 2524-6410,Vol.3, Issue 1 ,October 2015, PP 6-10.
[18] A.Nellai Murugan and R.Megala Special Class of Relaxed Cordial Graphs ,International Journal Emerging Technologies in Engineering Research, ISSN 2524-6410,Vol.3, Issue 1 ,October 2015, PP 11-14.
[19] A.Nellai Murugan and P. Iyadurai Selvaraj, Cycle and *Armed Cap Cordial graphs*, Global Scholastic Research Journal of Multidisciplinary , ISSN 2349-9397,Volume , Issue 11, October 2015, PP 1-14. ISRA 0.416
[20] A.Nellai Murugan and J.Shinny Priyanka , Path Related Extended Mean Cordial Graphs ,International Journal of Resent Advances in Multi-Disciplinary Research, ISSN 2350-0743,Vol.2, Issue 10 ,October 2015, PP 0836-0840. I F 1.005.
[21] A.Nellai Murugan and G.Devakiruba., Divisor cordial labeling of Book and $_{Cn}$ @ $_{K1,n.,}$ OUTREACH, A Multi-Disciplinary Refereed Journal, Volume VIII, 2015, Pp. 86-92.
[22] A. Nellai Murugan and P. Iyadurai Selvaraj, Path Related Cap Cordial Graphs, OUTREACH, A Multi- Disciplinary Refereed Journal, Volume VIII, 2015, Pp. 100-106.
[23] Nellai Murugan, and A.Meenakshi Sundari, Product Cordial Graph of Umbrella and C_4 @ $S_{n.,}$ OUTREACH , A Multi-Disciplinary Refereed Journal, Volume VIII, 2015, Pp. 113 – 119.
[24] Nellai Murugan, and V.Sripratha, Mean Square Cordial Labeling, OUTREACH, A Multi-Disciplinary Refereed Journal, Volume VIII, 2015, Pp. 125 – 131.
[25] A.Nellai Murugan and A.L.Poornima, Meanness of Special Class of Graph, OUTREACH, A Multi-Disciplinary Refereed Journal, Volume VIII, 2015, Pp. 140 – 145.
[26] A.Nellai Murugan and G.Esther, Mean Cordial Labeling of Star, Bi-Star and Wheel, OUTREACH , A Multi-Disciplinary Refereed Journal, Volume VIII, 2015, Pp. 155 – 160.
[27] A. Nellai Murugan and P. Iyadurai Selvaraj, Cycle & Armed Cap Cordial Graphs, International Journal of Mathematical Combinatorics, ISSN 1937 1055 , Volume II, June 2016, Pp. 144-152.

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